A non-perturbative time-dependent string configuration

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Non-perturbative String Cosmology:

β functions =homogenous in X^0, to all order in $\alpha^{'}$

1 Control on Quantum Fluctuations

- Bare action
  $S = \frac{1}{4\pi} \int d^2\xi \sqrt{\gamma} \Big\{ \frac{1}{\alpha'} \gamma^{ab} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + R^{(2)} \phi_{bare}(X^0) \Big\}$

- Partition function
  $Z[V] = \int \mathcal{D}[X] \exp\left(-S - \int V_\mu X^\mu\right) = \exp(-W[V])$

- Classical fields $X_c^\mu \sqrt{\gamma} = \delta W / \delta V_\mu$

- Legendre transform $\Gamma[X_c] = W[V] - \int d^2\xi \sqrt{\gamma} V_\mu X_c^\mu$

- Exact evolution equation with α'

$$
\begin{aligned}
\dot{\Gamma} &= \frac{1}{4\pi} \int d^2\xi \sqrt{\gamma} \left\{ \gamma^{ab} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right\} \\
&+ \frac{\eta_{\mu\nu}}{4\pi} \mathrm{Tr} \left\{ \gamma^{ab} \frac{\partial}{\partial \xi^a} \frac{\partial}{\partial \zeta^b} \left(\frac{\delta^2 \Gamma}{\delta X^\mu(\xi) \delta X^\nu(\zeta)} \right)^{-1} \right\}
\end{aligned}
$$

- Gradient expansion

$$
\begin{aligned}
\Gamma &= \frac{1}{4\pi} \int d^2\xi \sqrt{\gamma} \Big\{ \gamma^{ab} g_{\mu\nu}(X^0) \partial_a X^\mu \partial_b X^\nu \\
&\qquad + R^{(2)} \phi(X^0) \Big\}
\end{aligned}
$$

Evolution for the dilaton

$$g_{00}\dot{\phi} = -\frac{\Lambda^2}{8\pi R^{(2)}}\left(1+(D-1)\frac{g_{00}}{g_{ii}}\right) + \frac{\phi^{''}}{4g_{00}}\ln\left(1+\frac{2\Lambda^2 g_{00}}{R^{(2)}\phi^{''}}\right)$$

Λ= fixed world sheet cut off

Non-trivial $\alpha^{'}$-independent dilaton, $\dot{\phi}=0$:

g_{00} proportional to $\phi^{''}$
g_{00} proportional to g_{ii}

$$g_{00}\dot{\phi} = -\frac{\Lambda^2}{R^{(2)}}C_1 + \frac{C_2}{2}\ln\left(1+\frac{\Lambda^2}{R^{(2)}C_2}\right) = 0$$

$$\underline{g_{\mu\nu}(X^0) \propto \phi^{''}(X^0)\eta_{\mu\nu}}$$

$\longrightarrow$ condition for the dilaton to be independent of quantum fluctuations

2 Conformal properties

One-loop Weyl invariance conditions:

$$\begin{aligned} \beta^g_{\mu\nu} &= R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \mathcal{O}(\alpha^{'}) = 0 \\ \beta^\phi &= \frac{D-26}{6\alpha^{'}} - \frac{1}{2}\nabla^2\phi + \partial^\rho\phi\partial_\rho\phi + \mathcal{O}(\alpha^{'}) = 0 \end{aligned}$$

NOT satisfied by $g_{\mu\nu}(X^0) \propto \phi^{''}(X^0)\eta_{\mu\nu}$

BUT for the specific case

$$\underline{\phi = \phi_0 \ln(X^0)} \qquad\qquad \underline{g_{\mu\nu} = \frac{\kappa}{(X^0)^2}\eta_{\mu\nu}}$$

homogeneous beta-functions: to all orders in $\alpha^{'}$

$$\begin{aligned} \beta^g_{00} &= \frac{1}{(X^0)^2}\sum_{m=0}^{\infty}\xi_m\left(\frac{\alpha^{'}}{\kappa}\right)^m, \\ \beta^g_{ij} &= \frac{\delta_{ij}}{(X^0)^2}\sum_{m=0}^{\infty}\zeta_m\left(\frac{\alpha^{'}}{\kappa}\right)^m, \\ \beta^\phi &= \frac{1}{\alpha^{'}}\sum_{m=0}^{\infty}\eta_m\left(\frac{\alpha^{'}}{\kappa}\right)^m, \end{aligned}$$

Reparametrisation of the string (two loops)

$$\begin{aligned}\tilde{g}_{\mu\nu} &= g_{\mu\nu} + \alpha^{'} g_{\mu\nu}\left(b_1 R + b_2 \partial^\rho \phi \partial_\rho \phi + b_3 \nabla^2 \phi\right), \\ \tilde{\phi} &= \phi + \alpha^{'}\left(c_1 R + c_2 \partial^\rho \phi \partial_\rho \phi + c_3 \nabla^2 \phi\right),\end{aligned}$$

For the non-trivial fixed-point configuration:

$$\begin{aligned}\tilde{g}_{\mu\nu} &\propto g_{\mu\nu} \\ \tilde{\phi} &= \phi + \text{constant}\end{aligned}$$

$\longrightarrow$ no effect on the time-dependence

Effect on the beta functions:

$$\tilde{\beta}^i = \beta^i + (\tilde{g}^j - g^j)\frac{\partial \beta^i}{\partial g^j} - \beta^j \frac{\partial}{\partial g^j}(\tilde{g}^i - g^i)$$

$\longrightarrow$ Always possible to find $\tilde{\beta}^g_{\mu\nu} = \tilde{\beta}^\phi = 0$
for any target space dimension
and any dilaton amplitude

higher loops: similar procedure

Conjecture: Weyl invariance conditions satisfied
to all orders

3 Wilsonian properties

Running world sheet cut off k

$$\phi_k(X^0) \qquad \text{and} \qquad g_k^{\mu\nu}(X^0) = \kappa_k(X^0)\eta^{\mu\nu}$$

Exact renormalization group equation
(sharp cut off)

$$\begin{aligned} R^{(2)}\partial_k\phi_k(x^0) &= -(D-1)k\ln\left(\frac{\kappa_k(x^0)}{\kappa_k(1)}\right) \\ &\quad -k\ln\left(\frac{2\kappa_k(x^0)k^2 + \alpha' R^{(2)}\phi_k''(x^0)}{2\kappa_k(1)k^2 + \alpha' R^{(2)}\phi_k''(1)}\right) \end{aligned}$$

Exact solution

$$\begin{aligned} g_k^{\mu\nu}(X^0) &= \frac{A_k}{(X^0)^2}\eta^{\mu\nu} \\ \phi_k(X^0) &= \left(\phi_0 + \frac{Dk^2}{R^{(2)}}\right)\ln(X^0) \end{aligned}$$

limit $k \to 0$:

new α'-independent configuration

$\Updownarrow$

IR Wilsonian fixed point

4 Cosmological properties

In the Einstein Frame:

$$dt^2 - a^2(t)(d\vec{x})^2 = \exp\left(\frac{-4\phi(x^0)}{D-2}\right) g_{\mu\nu}(x^0)dx^\mu dx^\nu$$

Cosmic time: $t \propto (x^0)^{-\frac{2\phi_0}{D-2}}$

Scale factor: $a(t) \propto t^{1+\frac{D-2}{2\phi_0}}$

$\longrightarrow$ Minkowski space time for $D - 2 + 2\phi_0 = 0$

Dilaton in terms of cosmic time

$$\phi = -\frac{D-2}{2}\ln t$$

5 Quantization of target space dimension

Wick rotation in the target space: $X^0 \to iX^0$
Kinetic term:

$$\frac{\eta_{\mu\nu}}{(X^0)^2}\partial_a X^\mu \partial_b X^\nu \longrightarrow \frac{\delta_{\mu\nu}}{(X^0)^2}\partial_a X^\mu \partial_b X^\nu$$

Potential term:

$$R^{(2)}\phi_0 \ln(X^0) \longrightarrow R^{(2)}\phi_0 \left(\ln(X^0) + i\frac{\pi}{2}\right)$$

Action

$$S \longrightarrow S_E + i\frac{\pi}{2}\phi_0\chi$$

Partition function (spherical world sheet, $\chi = 2$):

$$Z \longrightarrow Z_E \times \exp\left(i\pi\phi_0\right)$$

Reality condition: $\phi_0 =$ integer
Minkowski target space condition: $D - 2 + 2\phi_0 = 0$

$\longrightarrow$ D=4,6,8,...